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METHODS OF COMPUTING THE STRENGTH AND STIFFNESS OF PLYWOOD STRIPS IN BENDING[1]

By

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INTRODUCTION

This report gives methods of computing the strength and stiffness of plywood strips in bending. The methods have been developed with a background of nearly 4,000 static bending tests, and are believed to be reasonably accurate. Further analysis of the data may, however, result in changes in some of the details.

The methods are limited in application to plywood having the grain of adjacent plies at 90° and having the grain of the face plies either parallel or perpendicular to the span. Data are not yet available for other cases.

STIFFNESS OF PLYWOOD STRIPS IN BENDING

The deflection of the center of a simply supported plywood strip under center loading will be given approximately by the equation

$$w = \frac{Pa^3}{48\,E_c I} \qquad (1)$$

where w = deflection of the center of the strip

P = load

a = span

E_c = apparent modulus of elasticity in bending

I = moment of inertia of the whole cross section of the strip about its central line (i.e., $I = bh^3/12$)

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[1]This is one of a series of progress reports prepared by the Forest Products Laboratory relating to the use of wood in aircraft. Results here reported are preliminary and may be revised as additional data become available.

b = width of strip

h = depth of strip

The apparent modulus of elasticity in bending, E_c, may be found by

$$E_c = \frac{\sum_{i=1}^{i=n} E_i I_i}{I} \qquad (2)$$

where E_i = modulus of elasticity of the i^{th} ply measured parallel to the span

I_i = moment of inertia of the cross section of the i^{th} ply about the neutral axis of the cross section

and the other terms are as defined above.

For plywood constructed symmetrically, that is, with pairs of plies of equal thicknesses at equal distances from the center, the neutral axis of the strip will be at the center of the plywood. This will usually be the case, at least for carefully made plywood; but occasions may arise when unsymmetrical plywood will be encountered. In such an event, the location of the neutral axis may be found by

$$d_{na} = \frac{\sum_{i=1}^{i=n} A_i E_i d_i}{\sum_{i=1}^{i=n} A_i E_i} \qquad (3)$$

where d_{na} = distance from top of beam to neutral axis

d_i = distance from top of beam to center of i^{th} ply

A_i = cross-sectional area of i^{th} ply

For plywood having all plies of equal thickness, for example, values of E_c will be given by use of equation (2), as:

$$\text{3-ply;}\quad E_1 = \frac{r_T + 26}{27} E_L \qquad E_2 = \frac{1 + 26r_T}{27} E_L$$

$$\text{5-ply;}\quad E_1 = \frac{26r_T + 99}{125} E_L \qquad E_2 = \frac{26 + 99r_T}{125} E_L \tag{4}$$

$$\text{7-ply;}\quad E_1 = \frac{99r_T + 244}{343} E_L \qquad E_2 = \frac{99 + 244r_T}{343} E_L$$

$$\text{9-ply;}\quad E_1 = \frac{244r_T + 485}{729} E_L \qquad E_2 = \frac{244 + 485r_T}{729} E_L$$

where $E_1 = E_c$ for plywood strips having the grain of the outer plies parallel to the span

$E_2 = E_c$ for plywood strips having the grain of the outer plies perpendicular to the span

E_L = modulus of elasticity of wood in the longitudinal direction (parallel to the grain)

$r_T = E_T/E_L$

E_T = modulus of elasticity of wood in the tangential direction

Equations (4) above apply strictly only to plywood made of rotary-cut veneer, since the ratio r_T has been used. For plywood made of quarter-sliced veneer, the ratio $r_R = E_R/E_L$ should be used, since the modulus of elasticity in the radial direction would be involved.

While equation (1) for a simply supported beam loaded in the center has been used as an example, values of E_c as found from equation (2) may be used in the appropriate equation for deflection corresponding to the method of loading, substituting E_c for E in the usual equations.

If the effect of shear deformation is to be taken into account, equation (1) (and the corresponding equations for other loadings) would be modified by another term, as, for example,

$$w = \frac{Pa^3}{48E_cI}\left(1 + e\,\frac{h^2}{a^2}\right) \tag{5}$$

where e = a term involving the elastic constants of wood and the construction of the plywood

h = depth of beam

a = span of beam

Published values of E_L based on static bending tests made at the Forest Products Laboratory are about 10 percent below the true values of E_L for the material because of the effect of shear deformation.

Where no correction for shear deformation is to be made, E_c may be calculated with the Forest Products Laboratory published values of E_L. Thus, values of E_c so calculated will give values of deflection correct for cases when the effect of shear deformation amounts to about 10 percent. Usually these values of E_c will be sufficiently exact.

Where, however, a correction for shear deformation is to be made, the published values of E_L should be increased 10 percent.

Some values for (e) are given in table 1. A discussion of their uses and limitations will be found in the discussion accompanying table 1.

STRENGTH OF PLYWOOD STRIPS IN BENDING

For Plywood Having the Grain of the Outer Plies Parallel to the Span

The strength of plywood strips of this type in bending will be given by

$$M = Kr_1 \frac{SI}{c} \qquad (3)$$

where M = resisting moment of plywood strip

K = constant

S = stress in outer fiber of outermost longitudinal ply

c = distance from neutral axis to outer fiber of outermost longitudinal ply

$r_1 = E_1/E_L$

For Plywood Having the Grain of the Outer Plies Perpendicular to the Span

The strength of plywood strips of this type in bending will be given by

$$M = Kr_m \frac{SI}{c} \qquad (7)$$

where $r_m = E_m/E_L$

$E_m = E_c$ for a strip of plywood having the outer plies at right angles to the span, but considering that the outer ply on the tension side does not act.

The position of the neutral axis for this case may be found by use of equation (3) except that there will be (n - 1) plies, the outer ply on the tension side being omitted from consideration in calculating E_m. (The value of I as before is based on the full cross section about its center line, including all plies.)

PLYWOOD MADE OF MORE THAN ONE SPECIES

The formulas previously presented in this report have been checked by tests of specimens made with all plies of the same species. A form of plywood commonly met in aircraft work is that made of two or more species. In the absence of test data to serve as a check, and pending the development of more accurate methods, the following procedure is suggested for calculating the bending strength and stiffness of plywood of mixed species:

In computing deflections, equation (2) for E_c may still be used, except that in this case the values of E_i for the parallel plies will not be the same for all plies, but will be those corresponding to the species. Similarly, the values of E_i for the perpendicular plies will be those for the species under consideration.

In computing strengths, equations (6) and (7) may still be used. The values of r will be determined from E_c as indicated above. The value of S, which for plywood made with all plies of the same species would be the same for either face grain direction, may now be different for the two directions, but in any case will correspond to the outermost ply whose grain direction is parallel to the span.

VALUES OF THE RATIOS $r_T = E_T/E_L$ and $r_R = E_R/E_L$

Values of E_T and E_R from which to compute the ratios r_T and r_R are available for a few species. The data are, however, incomplete in that only a few species have been tested and that little is known about the variation of these ratios with variations in specific gravity or with changes in moisture content.

Such data as are available are presented in "Elastic Properties of Wood," Forest Products Laboratory Report No. 1528 and supplements thereto. As more data become available, additional supplements will be issued. Data from these publications should be used when applicable. For cases in which no data are available, the use of values of $r_T = 0.05$ and $r_R = 0.10$ is suggested.

As pointed out later, even approximate values of these ratios will generally give results only slightly in error, so that lack of exact values is not a serious deterrent to the use of the proposed methods.

ERRORS INTRODUCED BY NEGLECTING TRANSVERSE PLIES

The transverse plies may, in general, be neglected with relatively small error except in the case of 3-ply plywood having the grain of the outer plies perpendicular to the span.

For plywood having the grain of the outer plies parallel to the span, such neglect will generally result in small errors. For plywood having the grain of the outer plies perpendicular to the span, however, the error is somewhat larger, and for 3-ply plywood of this type the error may be considerable. Figures 1 to 4 show the error that may be expected from neglect of the transverse plies for 3, 5, 7, and 9-ply plywood of two types for various values of r. While certain approximations were used in calculating the values for these figures, they will be useful as a guide in determining whether or not neglect of the transverse plies will be a serious matter. It will be recognized that interpolation in the curves of figures 1 to 4 is not possible. The connection of the plotted points by straight lines has been done only to emphasize the trend, as the number of plies is varied, of the ratios concerned.

The foregoing discussion also permits the conclusion that, for most cases, reasonably accurate results may be obtained even with assumed values of r. Therefore, reasonable results may be obtained for species for which no values of r_T and r_R are available by using the approximate values previously given as a guide in estimating values for other species.

VALUES FOR (e) FOR USE IN EQUATION (5)

The factor (e) for use in correcting deflections for the deformations due to shear has been evaluated for 3, 5, 7, and 9-ply Douglas-fir and Sitka spruce plywood having all plies the same thickness for the case of a simply

supported beam under center load. Corresponding values for other cases have not been evaluated. Values of (e) for a uniformly loaded, simply supported beam may be expected to be slightly smaller than for a simply supported beam with center load, but no exact values are available.

It should be noted, however, that reasonable corrections for shear deformations may be made even with values of (e) considerably in error. Assume, for example, that the proper shear correction was (1 + 0.10) and that the value of (e) used was 20 percent low. The computed correction would then be (1 + 0.08), which is only about 2 percent in error, so that the corrected deflection would be only about 2 percent in error.

From this it appears that the values of (e) given in table 1 may be used as approximations of the correct values for other types of plywood and plywood of other species, and may also be used to estimate approximately the shear corrections for such cases. It is expected that these values, for the types of plywood specified in table 1, will give corrections somewhat too large if used for the case of a uniformly loaded, simply supported beam.

Calculations based on the values in table 1 indicate that the correction for shear deformation will be less than 10 percent in practically all cases if a ratio of span to beam depth greater than 48 to 1 is maintained for plywood strips having the grain of the outer plies parallel to the span, and greater than 24 to 1 for plywood strips having the grain of the outer plies perpendicular to the span.

Table 1.--Values of (e) for plywood having all plies the same thickness and used as a simply supported beam with center load

No. of plies	(e)			
	Sitka spruce		Douglas-fir	
	Outer plies parallel to span	Outer plies perpendicular to span	Outer plies parallel to span	Outer plies perpendicular to span
3	184.8	8.6	141.7	9.9
5	160.8	46.3	118.4	36.5
7	147.1	63.5	108.5	48.8
9	138.8	73.3	102.5	55.7

VALUES OF K FOR USE IN EQUATIONS (6) AND (7)

The values of K suggested for use in equations (6) and (7) are:

Stresses at or below proportional limit --
  Outer plies parallel to span, K = 0.85
  Outer plies perpendicular to span, K = 0.90

Stresses at or near modulus of rupture --
  Outer plies parallel to span, K = 0.85
  Outer plies perpendicular to span,
    3-ply, K = 1.10
    Others, K = 0.95

If the approximate method, neglecting the effect of the transverse plies, is used, the values of K for the outer perpendicular plywood will have to be increased somewhat, since this approximation results in values of moment lower than the true values. The difference will, of course, depend upon the construction of the plywood, but an approximation of the correct factor may be obtained by dividing the appropriate factor above by the appropriate ratio given in figure 2 or 4.

For outer-parallel strips the approximate method gives results, in general, close enough to those of the exact method so that no modification of the factor is necessary.

STRESS VALUES TO BE USED IN CALCULATIONS

The stress values or modulus of elasticity values to be used in the calculations will depend upon the use for which the plywood is intended.

U. S. Department of Agriculture Technical Bulletin 479, "Strength and Related Properties of Woods Grown in the United States," gives average strength and modulus of elasticity values for the more common species as found from tests on small clear specimens.

U. S. Department of Agriculture Miscellaneous Publication 185, "Guide to the Grading of Structural Timbers and the Determination of Working Stresses," gives basic stress values for determination of working stresses for the more common structural species.

ANC Bulletin 18, "Design of Wood Aircraft Structures," gives values for a number of species for use in aircraft design.

The appropriate design values given in the sources above are applicable to calculations on plywood; that is, the appropriate values of stress or modulus of elasticity will be substituted for S or E_L in the equations given in this report.

SUMMARY OF NOTATION

a = span of beam

A_i = cross-sectional area of i^{th} ply

b = width of beam

c = distance from neutral axis to outer fiber of outermost longitudinal ply

d_{na} = distance from top of beam to neutral axis

d_i = distance from top of beam to center of i^{th} ply

e = a term dependent upon the elastic constants of wood and the construction of the plywood

E_c = apparent modulus of elasticity in bending

E_1 = E_c for plywood strips having the grain of the outer plies parallel to the span

E_2 = E_c for plywood strips having the grain of the outer plies perpendicular to the span

E_L = modulus of elasticity of wood in the longitudinal direction (parallel to the grain)

E_T = modulus of elasticity of wood in the tangential direction

E_R = modulus of elasticity of wood in the radial direction

E_i = modulus of elasticity of the i^{th} ply measured parallel to the span

E_m = E_c for a strip of plywood having the grain of the outer plies at right angles to the span, but considering that the outer ply on the tension side does not exist

h = depth of beam

I = moment of inertia of the whole cross section of the strip about its central line (i.e., $I = bh^3/12$)

I_i = moment of inertia of the cross section of the i^{th} ply about the neutral axis of the cross section

K = constant

M = resisting moment of plywood strip

n = number of plies

P = load

$r_T = E_T/E_L$

$r_R = E_R/E_L$

$r_l = E_l/E_L$

$r_m = E_m/E_L$

S = stress in outer fiber of outermost longitudinal ply

w = deflection of the center of the strip

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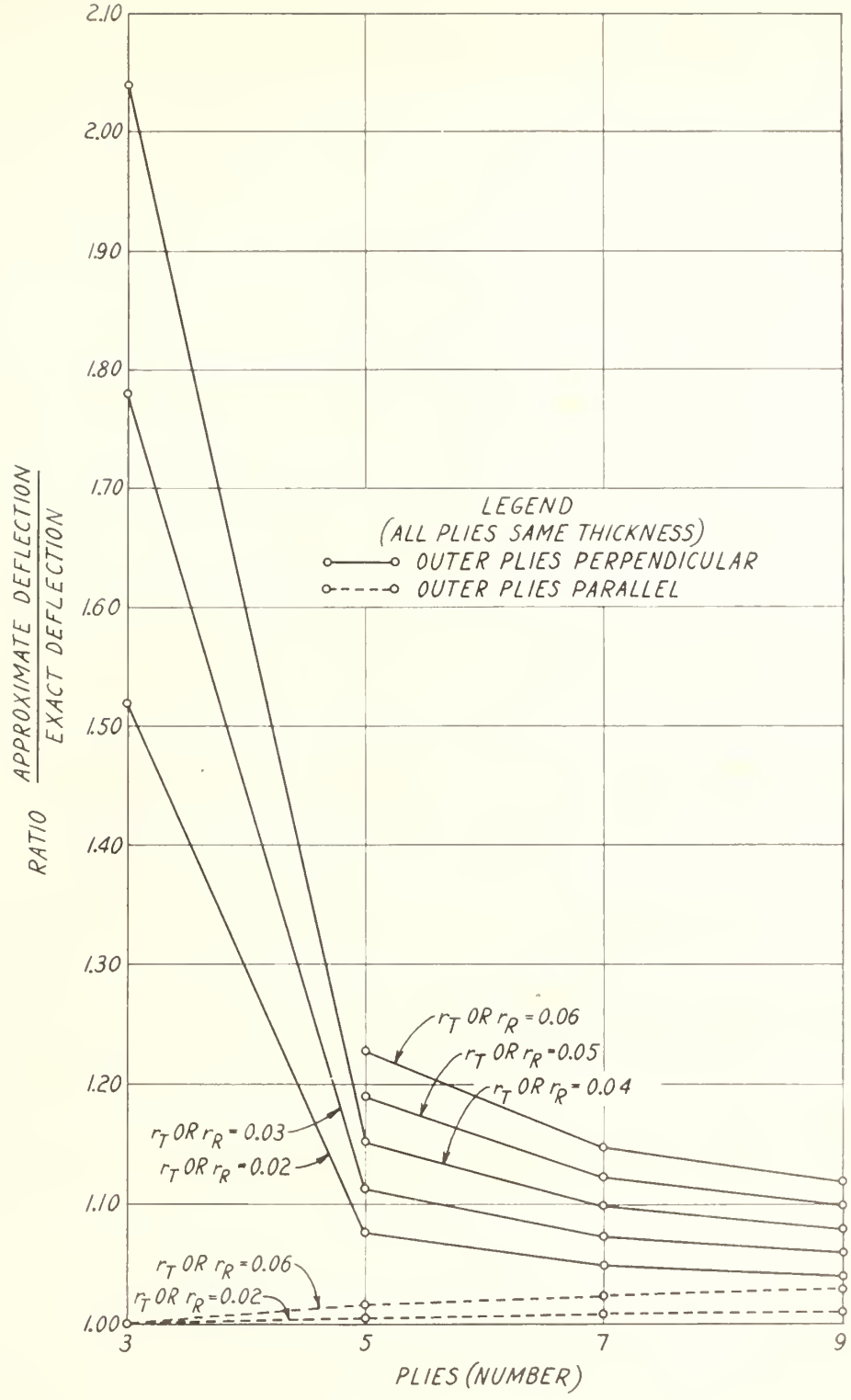

FIG. 1
RATIOS OF DEFLECTIONS CALCULATED
BY NEGLECTING TRANSVERSE PLIES TO DEFLECTIONS
CALCULATED BY CONSIDERING TRANSVERSE PLIES.

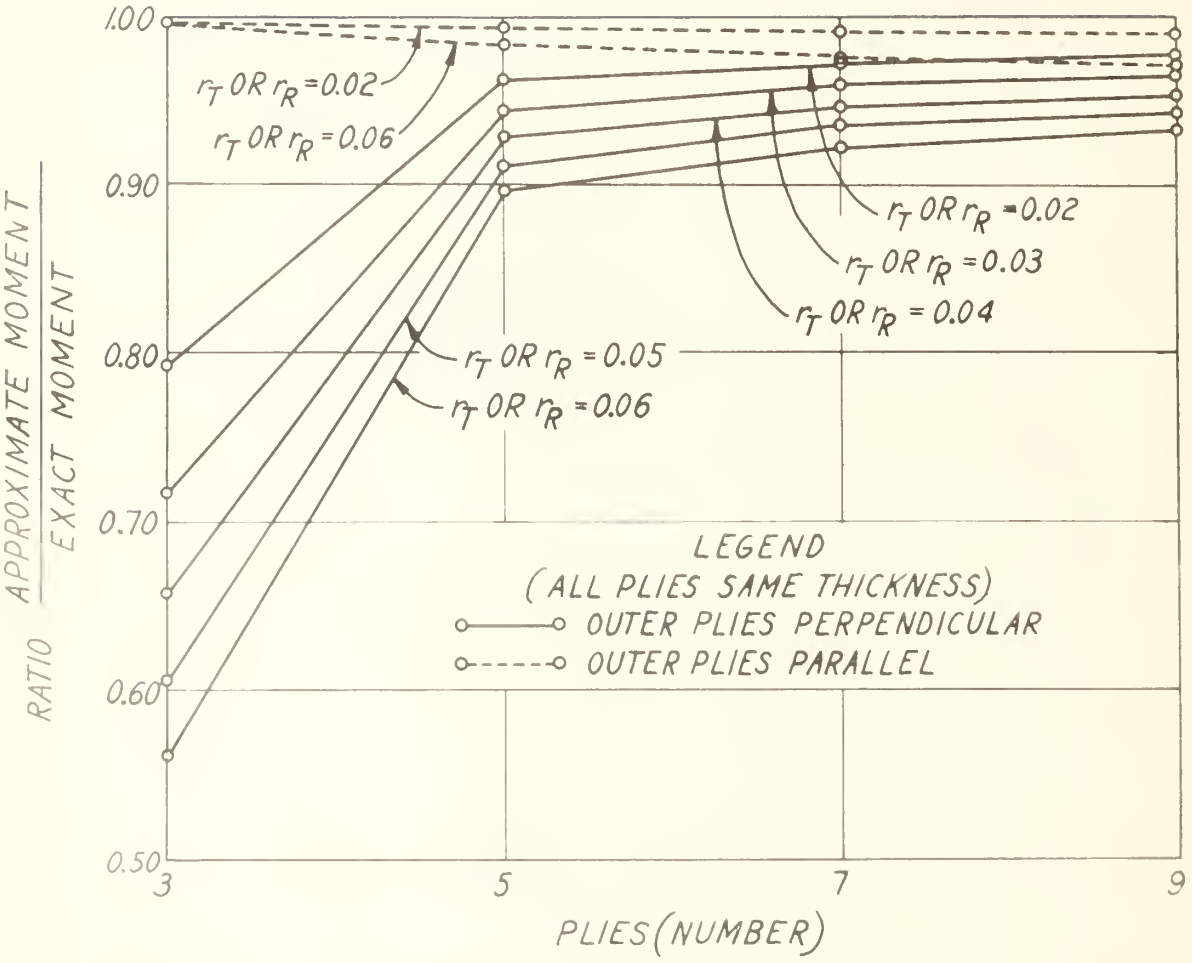

FIG. 2
RATIOS OF MOMENTS CALCULATED
BY NEGLECTING TRANSVERSE PLIES TO MOMENTS
CALCULATED BY CONSIDERING TRANSVERSE PLIES.

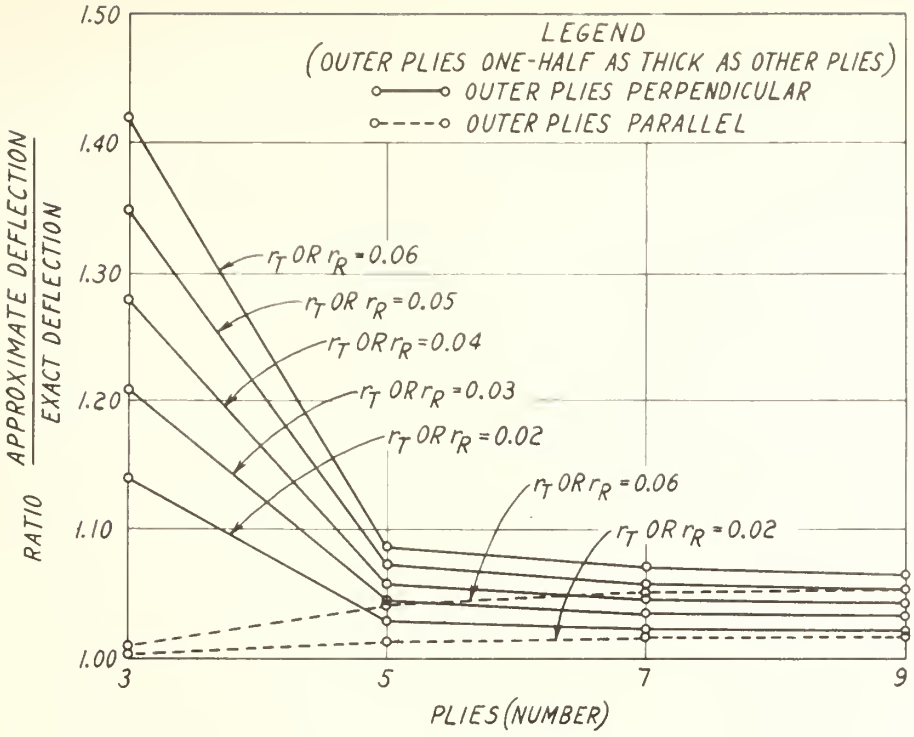

FIG. 3
RATIOS OF DEFLECTIONS CALCULATED
BY NEGLECTING TRANSVERSE PLIES TO DEFLECTIONS
CALCULATED BY CONSIDERING TRANSVERSE PLIES.

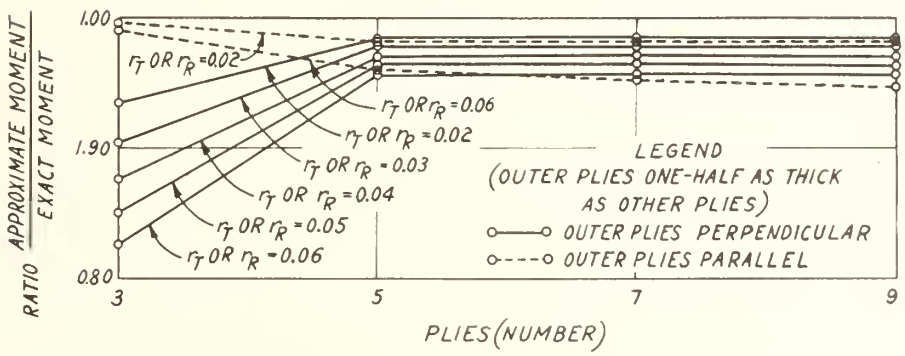

FIG. 4
RATIOS OF MOMENTS CALCULATED
BY NEGLECTING TRANSVERSE PLIES TO MOMENTS
CALCULATED BY CONSIDERING TRANSVERSE PLIES.

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